

Método Runge-Kutta - EDO

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Exercício utilizando o método RK4

$$\frac{dy}{dx} = x^2 - y$$

$$x_0 = 0 \rightarrow x_f = 0.4$$

$$y_0 = 1$$

$$h = 0.2$$

1º - Calcular o número de passos

$$n = \frac{x_f - x_0}{h} = \frac{0.4 - 0}{0.2} = 2 \text{ passos}$$

→ $x = 0$ até $x = 0.2$

$x = 0.2$ até $x = 0.4$

Passo 1 → de $x=0$ até $x=0.2$ com $y_0 = 1$

$$K_1 = f(x_n, y_n)$$

$$K_1 = 0^2 - 1 = -1$$

$$K_2 = f\left(x_n + \frac{h}{2}, y_n + \frac{h}{2} \cdot K_1\right)$$

$$K_2 = \left(0 + \frac{0.2}{2}\right)^2 - \left(1 + \frac{0.2}{2} \cdot (-1)\right)$$

$$K_2 = 0.1^2 - 0.9 = -0.89$$

$$K_3 = f\left(x_n + \frac{h}{2}, y_n + \frac{h}{2} \cdot K_2\right)$$

$$K_3 = \left(0 + \frac{0.2}{2}\right)^2 - \left(1 + \frac{0.2}{2} \cdot (-0.89)\right)$$

$$K_3 = 0.1^2 - 0.911 = -0.901$$

$$K_4 = f(x_n + h, y_n + h \cdot K_3)$$

$$K_4 = (0 + 0.2)^2 - (1 + 0.2 \cdot (-0.901))$$

$$K_4 = 0.2^2 - 0.8198 = -0.7798$$

$$y_{n+1} = y_n + \frac{h}{6} (K_1 + 2K_2 + 2K_3 + K_4)$$

$$y_1 = 1 + \frac{0.2}{6} (-1 + 2(-0.89) + 2(-0.901) + (-0.7798))$$

$$y_1 = 1 + \frac{0.2}{6} (-1 + (-1.78) + (-1.802) + (-0.7798)) \approx 0.8213$$

Passo 2 → de $x = 0,2$ até $x = 0,4$ com $y_1 = 0,8213$

$$K_1 = f(x_n, y_n)$$

$$K_1 = (0,2)^2 - 0,8213 = -0,7813$$

$$K_2 = f\left(x_n + \frac{h}{2}, y_n + \frac{h}{2} \cdot K_1\right)$$

$$K_2 = (0,2 + \frac{0,2}{2})^2 - (0,8213 + \frac{0,2}{2} \cdot (-0,7813))$$

$$K_2 = 0,3^2 - 0,7432 = -0,6532$$

$$K_3 = f\left(x_n + \frac{h}{2}, y_n + \frac{h}{2} \cdot K_2\right)$$

$$K_3 = (0,2 + \frac{0,2}{2})^2 - (0,8213 + \frac{0,2}{2} \cdot (-0,6532))$$

$$K_3 = 0,3^2 - 0,75598 = -0,66598$$

$$K_4 = f(x_n + h, y_n + h \cdot K_3)$$

$$K_4 = (0,2 + 0,2)^2 - (0,8213 + 0,2 \cdot (-0,66598))$$

$$K_4 = 0,4^2 - 0,6881 = -0,5281$$

$$y_{n+1} = y_n + \frac{h}{6} (K_1 + 2K_2 + 2K_3 + K_4)$$

$$y_2 = 0,8213 + \frac{0,2}{6} (-0,7813 + 2(-0,6532) + 2(-0,66598) + (-0,5281))$$

$$y_2 = 0,8213 + \frac{0,2}{6} (-0,7813 + (-1,3064) + (-1,33196) + (-0,5281))$$

$$y_2 = 0,8213 + (-0,1316) = 0,6897$$

Exercício Runge Kutta

Usando

$$y_{n+1} = y_n + \frac{h}{2} [k_1 + k_2], \quad n = 0, 1, 2, \dots$$

$$k_1 = f(x_n, y_n)$$

$$k_2 = f(x_n + h, y_n + h k_1)$$

$$\begin{cases} y' = -y + x + 2 \\ y(0) = 2 \end{cases} \quad x \in [0, 0,3], \quad h = 0,1.$$

$$f(x, y) = -y + x + 2; \quad h = 0,1; \quad x_0 = 0 \text{ e } y_0 = 2$$

$$n = 1, 2, 3 \quad x_1 = 0,1; \quad x_2 = 0,2 \quad \text{e} \quad x_3 = 0,3.$$

Então, para $n = 0$:

$$y_1 = y_0 + \frac{h}{2} [k_1 + k_2]$$

$$k_1 = f(x_0; y_0) = f(0; 2) = -2 + 0 + 2 = 0,000$$

$$k_2 = f(x_0 + h; y_0 + h k_1) = f(0,1; 2) = -2 + 0,1 + 2 =$$

$$0,100$$

$$\rightarrow y_1 = 2 + \frac{0,1}{2} [0,000 + 0,100] = 2,005$$

Para $n = 1$

$$y_2 = y_1 + \frac{h}{2} [k_1 + k_2]$$

$$k_1 = f(x_1, y_1) = f(0, 1; 2, 0050) = -2,0050 + 0,1 + 2 = 0,0950$$

$$k_2 = f(x_1 + h; y_1 + hk_1) = f(0, 2; 2, 014501) = -2,0145 + 0,2 + 2 = 0,1855$$

$$\rightarrow y_2 = 2,0050 + \frac{0,1}{2} [0,0950 + 0,1855] = 2,0190$$

Para $n = 2$

$$y_3 = y_2 + \frac{h}{2} [k_1 + k_2]$$

$$k_1 = f(x_2, y_2) = f(0, 2; 2, 0190) = -2,0190 + 0,2 + 2 = 0,1809$$

$$k_2 = f(x_2 + h; y_2 + hk_1) = f(0, 3; 2, 0371) = -2,0371 + 0,3 + 2 = 0,2628$$

$$y_3 = 2,0190 + \frac{0,1}{2} [0,1809 + 0,2628] = 2,0412$$

K	x_n	y_k	$y(x_k)$
0	0,0000	2,0000	2,0000
1	0,1000	2,0050	2,0048
2	0,2000	2,0190	2,0187
3	0,3000	2,0412	2,0408

Example Método RK4

dados:

$$x_0 = 0$$

$$y_0 = 1$$

$$h = 0.1$$

$$f(x, y) = x + y$$

$$K_1 = f(x_n, y_n)$$

$$K_1 = f(0, 1)$$

$$K_1 = 0 + 1$$

$$K_1 = 1,$$

$$K_2 = f\left(x_n + \frac{h}{2}, y_n + \frac{h}{2} K_1\right)$$

$$K_2 = f\left(0 + \frac{0.1}{2}; 1 + \frac{0.1}{2} \cdot 1\right)$$

Resposta:

$$y(0.1) \approx 1.11034$$

$$K_2 = f(0.05; 1 + 0.05)$$

$$K_2 = f(0.05; 1.05)$$

$$K_2 = 0.05 + 1.05$$

$$K_2 = 1.1,$$

$$K_3 = f\left(x_n + \frac{h}{2}, y_n + \frac{h}{2} \cdot K_2\right)$$

$$K_3 = f\left(0 + \frac{0.1}{2}; 1 + \frac{0.1}{2} \cdot 1.1\right)$$

$$K_3 = f(0.05; 1.055)$$

$$K_3 = 0.05 + 1.055$$

$$K_3 = 1.105,$$

$$K_4 = f(x_n + h; y_n + h K_3)$$

$$K_4 = f(0 + 0.1; 1 + 0.1 \cdot 1.105)$$

$$K_4 = f(0.1; 1.1105)$$

$$K_4 = 0.1 + 1.1105$$

$$K_4 = 1.2105,$$

$$y_{n+1} = y_n + \frac{h}{6} (K_1 + 2K_2 + 2K_3 + K_4)$$

$$y_1 = 1 + \frac{0.1}{6} (1 + 2 \cdot 1.1 + 2 \cdot 1.105 + 1.2105)$$

$$y_1 = 1 + \frac{0.1}{6} (6.6205)$$

$$y_1 \approx 1.11034$$

RK4 - questão para apresentação

Adensamento de uma fundação - Taxa de recalque do solo

$$\frac{ds}{dt} = 0,8s + 2t$$

$$S(0) = 0 \text{ mm}$$

$$t = 0 \rightarrow t = 2h$$

$$h = 1h$$

$$n = \frac{t_F - t_i}{h} = \frac{2 - 0}{1} = 2 \text{ passos} //$$

$$K_1 = f(t_n, S_n)$$

$$K_2 = f\left(t_n + \frac{h}{2}, S_n + \frac{h}{2} K_1\right)$$

$$K_3 = f\left(t_n + h, S_n + h K_2\right)$$

$$K_4 = f(t_n + h, S_n + h K_3)$$

$$S_{n+1} = S_n + \frac{h}{6} (K_1 + 2K_2 + 2K_3 + K_4)$$

1ª iteração $\rightarrow t=0$ até $t=1$

$$K_1 = 0,8(0) + 2(0) = 0$$

$$K_2 = 0,8\left(0 + \frac{1}{2} \cdot 0\right) + 2\left(0 + \frac{1}{2}\right) = 1$$

$$K_3 = 0,8\left(0 + \frac{1}{2} \cdot 1\right) + 2\left(0 + \frac{1}{2}\right) = 1,4$$

$$K_4 = 0,8(0 + 1 \cdot 1,4) + 2(0 + 1) = 3,12$$

$$S_{n+1} = 0 + \frac{1}{6} (0 + 2(1) + 2(1,4) + 3,12) = 1,32 \text{ mm} //$$

• $S = 1,32 \text{ mm}$ $t = 1$

$$K_1 = 0,8(1,32) + 2(1) = 3,056$$

$$K_2 = 0,8\left(1,32 + \frac{1}{2} \cdot 3,056\right) + 2\left(1 + \frac{1}{2}\right) = 5,2784$$

$$K_3 = 0,8\left(1,32 + \frac{1}{2} \cdot 5,2784\right) + 2\left(1 + \frac{1}{2}\right) \approx 6,1674$$

$$K_4 = 0,8(1,32 + 1 \cdot 6,1674) + 2(1 + 1) \approx 9,9899$$

$$S_{n+1} = 1,32 + \frac{1}{6} (3,056 + 2(5,2784) + 2(6,1674) + 9,9899) \approx 7,3096 \text{ mm} \rightarrow 7,31 \text{ mm} //$$